(10) Trigonometry
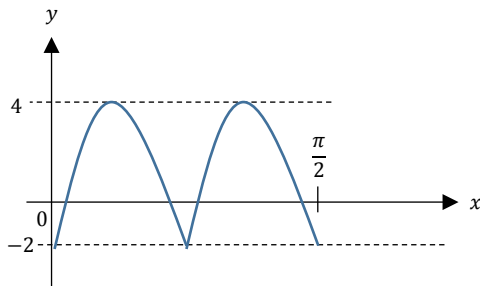
1. Find all the values of x between -3 and 2 for which $\sec\frac{\pi}{4}\sec^2(2x-2) = -\tan\frac{5\pi}{3}$

2. Find all the angles between 0° and 180° for the equation $\cos 3x = -11\cos^2 x$.

3. Solve the equation $\cos^2\theta + 3\sin\theta\cos\theta = -1$ for $0^o \le \theta \le 180^o$

4. a) Given that $\cos 2x = a + b$ and $\sin 2x = a - b$. Show that $a^2 + b^2 = \frac{1}{2}$.

   b) Find a function that has the following graph

5. Prove that $\sin^4 A + \cos^4 A = \frac{1}{4}(3 + \cos 4A)$.

   $(Hint: Square(\sin^2 A + \cos^2 A)\ to\ help\ proof\ above\ equation)$

6. ABC is a triangle where $\tan\left(\frac{\angle A}{2}\right) = \frac{1}{2}$.

   i) Show that $\tan\angle A = \frac{4}{3}$.

   ii) Find the exact values of $\sin(\angle B + \angle C)$ and $\cos(\angle B + \angle C)$

   (Hint: $\angle A + \angle B + \angle C = 180^o$)

7. The depth of water, y meters, at a particular coast, t hours after 12 am is given by:

   $y = 4 + 3\sin\left(\frac{\pi}{6}t\right)$, where $0 \le t \le 24$

   i) State the amplitude of y

   ii) What are the depths of water at high tide and low tide?

   iii) At what times of the day will low tide occur?

8. a) Prove the identity $\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}$

   b) Prove the identity $\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x \cos x)$

9. i) Prove the identity $\sin 2x - \tan x = \tan x \cos 2x$.

    ii) Hence, without using a calculator, find the value of $\tan(67.5^o)$.

10.
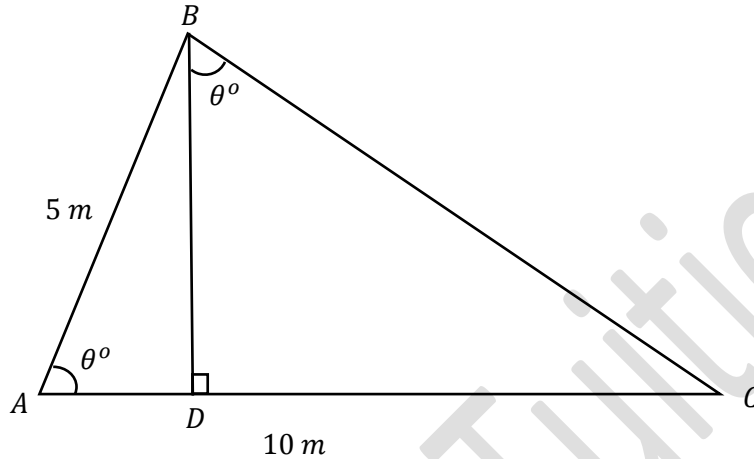
In the diagram above, ABC is a triangle such that $\angle BAD = \angle DBC = \theta^o$, $AB = 5\ m$ and $AC = 10\ m$.

i) Find BC (Leave your answer in surd form)

ii) Show that $BD = 5 \sin \theta^o$

iii) Show that $BD = 5\sqrt{5} \cos \theta^o$

iv) Hence, show that $2BD = 10 \cos(\theta^o - 24.1^o)$